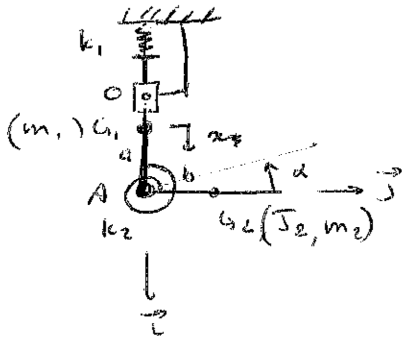


pas de pesanteur

$$\overrightarrow{OG_2} = \overrightarrow{OA} + \overrightarrow{AG_2} = (a + x)\,\vec{i} + b\,(-\sin\alpha\,\vec{i} + \cos\alpha\,\vec{j})$$

$$= (a + x - b\sin\alpha)\,\vec{i} + b\cos\alpha\,\vec{j}$$

$$\frac{d\overrightarrow{OG_2}}{dt} = (\dot{x} - b\cos\alpha\,\dot{\alpha})\,\vec{i} - b\sin\alpha\,\dot{\alpha}\,\vec{j}$$

$$T = \frac{1}{2} m_1 \dot{x}^2 + \frac{1}{2} m_2 \left[(\dot{x} - b\cos\alpha\,\dot{\alpha})^2 + (b\sin\alpha\,\dot{\alpha})^2\right]$$
$$+ \frac{1}{2} J_2 \dot{\alpha}^2$$

$$\frac{\partial T}{\partial \dot{x}} = m_1 \dot{x} + m_2 \dot{x} - b\cos\alpha\, m_2 \dot{\alpha}$$

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{x}} = (m_1 + m_2)\ddot{x} - b\cos\alpha\, m_2 \ddot{\alpha} + b\sin\alpha\, m_2 \dot{\alpha}^2$$

$$-\frac{\partial T}{\partial x} = 0$$

$$\frac{\partial T}{\partial \dot{\alpha}} = m_2 b^2 \dot{\alpha} - m_2 b\cos\alpha\,\dot{x} + J_2 \dot{\alpha}$$

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{\alpha}} = (J_2 + m_2 b^2)\ddot{\alpha} - m_2 b\cos\alpha\,\ddot{x} + m_2 b\sin\alpha\,\dot{\alpha}\dot{x}$$

$$-\frac{\partial T}{\partial \alpha} = -m_2 b\sin\alpha\,\dot{\alpha}\dot{x}$$

$$V = \frac{1}{2} k_1 x_1^2 + \frac{1}{2} k_2 \alpha^2$$

$$-\frac{\partial V}{\partial x} = -k_1 x$$

$$-\frac{\partial V}{\partial \alpha} = -k_2 \alpha$$

$$\begin{bmatrix} \cancel{-m_2 b}\cos\alpha & J_2 + m_2 b^2 \\ & \end{bmatrix} \begin{bmatrix} \ddot{x} \\ \ddot{\alpha} \end{bmatrix} + \begin{bmatrix} k_2 & 0 \\ & \end{bmatrix} \begin{bmatrix} x \\ \alpha \end{bmatrix}$$

$$\begin{bmatrix} (m_1+m_2) & -b\, m_2\cos\alpha \\ -m_2 b\cos\alpha & J_2+m_2b^2\end{bmatrix}\begin{bmatrix}\ddot x\\ \ddot\alpha\end{bmatrix} + \begin{bmatrix} b\sin\alpha\, m_2\,\dot\alpha^2 \\ m_2 b \sin\alpha\,\dot\alpha\,\dot x\end{bmatrix} = \begin{bmatrix}-k_1 & 0\\ 0 & -k_2\end{bmatrix}\begin{bmatrix}x\\ \alpha\end{bmatrix}$$

Pour α, x ~~petits~~ petits
$\dot\alpha$, $\dot x$ petits

$$\begin{bmatrix} m_1+m_2 & -b m_2 \\ -b m_2 & J_2+m_2b^2\end{bmatrix}\begin{bmatrix}\ddot x\\ \ddot\alpha\end{bmatrix} + \begin{bmatrix}k_1 & 0\\ 0 & k_2\end{bmatrix}\begin{bmatrix}x\\ \alpha\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}$$

Posons ~~$\tilde m = m_2/m_1$~~ $\tilde J = J_2$
~~$y = b\alpha$~~

$$\begin{bmatrix} \cancel{m_1(1+\tilde m)} & \cancel{-m_1\tilde m\, b} \\ \cancel{-m_1\tilde m} & \cancel{J_2+m_1\tilde m\, b}\end{bmatrix}$$

Posons $\tilde x = \dfrac{x}{b}$

$$\begin{bmatrix} b(m_1+m_2) & -b^2 m_2 \\ -b^2 m_2 & J_2+m_2b^2\end{bmatrix}\begin{bmatrix}\ddot{\tilde x}\\ \ddot\alpha\end{bmatrix} + \begin{bmatrix}b k_1 & 0\\ 0 & k_2\end{bmatrix}\begin{bmatrix}\tilde x\\ \tilde\alpha\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}$$

$J_2 = \tilde J\,(m_2 b^2)$

$$\begin{bmatrix} b(m_1+m_2) & -b m_2 \\ -1 & \tilde J+1\end{bmatrix}\begin{bmatrix} \\ \end{bmatrix} + \begin{bmatrix}b k_1 & 0\\ 0 & \frac{k_2}{m_2b^2}\end{bmatrix}\begin{bmatrix}\tilde x\\ \tilde\alpha\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}$$

$\tilde m = m_1/m_2$

$$\begin{bmatrix} \tilde m+1 & -1 \\ -1 & \tilde J+1\end{bmatrix}\begin{bmatrix}\ddot{\tilde x}\\ \ddot\alpha\end{bmatrix} + \begin{bmatrix}\frac{k_1}{m_2} & 0\\ 0 & \frac{k_2}{m_2b^2}\end{bmatrix}\begin{bmatrix}\tilde x\\ \tilde\alpha\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}$$

~~$\tilde k = \frac{k_1}{m_2}$~~

$\omega_0^2 = \dfrac{k_1}{m_2}$ $\qquad \tilde k = \dfrac{k_2}{b^2 k_1}$ $\qquad$ N/kg m $\qquad \dfrac{k_2}{b^2} = \tilde k\, k_1$

$$\begin{bmatrix} \tilde m+1 & -1 \\ -1 & \tilde J+1\end{bmatrix}\begin{bmatrix}\ddot{\tilde x}\\ \ddot\alpha\end{bmatrix} + \begin{bmatrix}\omega_0^2 & 0\\ 0 & \tilde k\,\omega_0^2\end{bmatrix}\begin{bmatrix}\tilde x\\ \tilde\alpha\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}$$

Détermination de la première fréquence propre.

cherchons $\begin{bmatrix} \tilde{x} \\ \alpha \end{bmatrix} = \begin{bmatrix} X \\ A \end{bmatrix} \sin \tilde{\omega}\, \omega_0 t$

$$\left| -\tilde{\omega}^2 \omega_0^2 \begin{bmatrix} \tilde{m}+1 & -1 \\ -1 & \tilde{J}+1 \end{bmatrix} + \begin{bmatrix} \omega_0^2 & 0 \\ 0 & \tilde{k}\,\omega_0^2 \end{bmatrix} \right| = 0$$

$$\begin{vmatrix} -\tilde{\omega}^2(\tilde{m}+1) + 1 & +\tilde{\omega}^2 \\ +\tilde{\omega}^2 & -\tilde{\omega}^2(\tilde{J}+1) + \tilde{k} \end{vmatrix} = 0$$

$$\tilde{\omega}^4 \left[(\tilde{m}+1)(\tilde{J}+1) \right] + \tilde{\omega}^2 \left[-(\tilde{J}+1) - (\tilde{m}+1) \right] + \tilde{k} - 1 - \tilde{\omega}^4 = 0$$

~~$\Delta = b^2 - 4ac$~~

$$\tilde{\omega}^4 \left[\tilde{m}\tilde{J} + \tilde{m} + \tilde{J} + 1 - 1 \right] + \tilde{\omega}^2 \left[-\tilde{J} - \tilde{m} - 2 \right] + \tilde{k} - 1 = 0$$

$$\left| \begin{array}{l} \tilde{k} = 2 \\ \tilde{m} = 1 \\ \tilde{J} = 1 \end{array} \right. \qquad \left| \begin{array}{l} \tilde{k} = 2 \\ \tilde{m} = 1 \\ \tilde{J} = 2 \end{array} \right.$$

$$3\,\tilde{\omega}^4 + \tilde{\omega}^2 \left[-4 \right] + 1 = 0$$

$\Delta = 16 - 4 \times 4$

$= 16 - 4 \times 3$

$= 4$

$\Delta = 2$

$$5\,\tilde{\omega}^4 + \tilde{\omega}^2(-5) + 1 = 0$$

$\Delta = 25 - 4 \times 5 \times 1$

$= 5$

$$\begin{cases} \tilde{\omega}_1 = \dfrac{5 - \sqrt{5}}{10} = \dfrac{5 - \sqrt{5}}{10} \simeq 0{,}276 \\ \tilde{\omega}_2 = \dfrac{5 + \sqrt{5}}{10} = 0{,}724 \end{cases}$$

Vecteurs propres

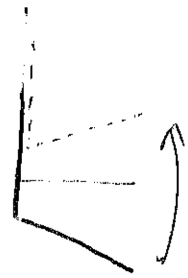
• à $\tilde{\omega}_1 \rightarrow \vec{V}_1 = \begin{bmatrix} X_1 \\ A_1 \end{bmatrix}$

$$\left[-\tilde{\omega}_1^2(\tilde{m}+1)+1\right] X_1 + \tilde{\omega}_1^2 A_1 = 0$$

$$\left[-\frac{10 - 2\sqrt{5}}{10} + 1\right] X_1 + \frac{5-\sqrt{5}}{10} A_1 = 0$$

$$A_1 = \frac{\cancel{10}}{5-\cancel{\sqrt{5}}}\left[-2\frac{\sqrt{5}}{\cancel{10}}\right] X_1$$

$$A_1 = -\frac{2}{\sqrt{5}-1} X_1$$



• à $\tilde{\omega}_2 \rightarrow \vec{V}_2 \begin{bmatrix} X_2 \\ A_2 \end{bmatrix}$

$$\left[-\omega_2^2(\tilde{m}+1)+1\right] X_2 + \tilde{\omega}_2^2 A_2 = 0$$

$$\left[-\frac{10+2\sqrt{5}}{10}+1\right] X_2 + \frac{5+\sqrt{5}}{10} A_2 = 0$$

$$A_2 = +\frac{\cancel{10}}{\underset{\sqrt{5}+1}{5+\sqrt{5}}}\left[+\frac{2\sqrt{5}}{\cancel{10}}\right] X_2$$

$$\boxed{A_2 = \frac{2}{\sqrt{5}+1} X_1}$$

Avec un oiseau en G_2

l'inertie de rotation du solide $2/G_2$ n'est pas changée car l'oiseau reste vertical

la masse du solide 2 est modifiée par la masse de l'oiseau m_0

$$m_2 \to m_2 + m_0$$

si $m_0 = m_2$ $\to$ $\tilde{m} : 1 \longrightarrow \frac{1}{2}$

$$\tilde{J} = 2$$

$$\tilde{k} = 2 \qquad \omega_0^2 \to \left[\frac{\omega_0}{2}\right]$$

la 1ère pulsation propre adim. est donnée par.

$$\left[\frac{1}{2}2 + \frac{1}{2} + 2\right]\tilde{\omega}^4 + \tilde{\omega}^2\left[-2 - \frac{1}{2} - 2\right] + 1 = 0$$

$$\frac{7}{2}\tilde{\omega}^4 + \tilde{\omega}^2\left(-\frac{9}{2}\right) + 1 = 0$$

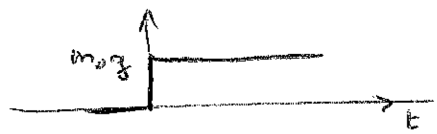
$$\Delta = b^2 - 4ac = \frac{81}{4} - 4\,\frac{7}{2} = \frac{81-56}{4} = \frac{25}{4}$$

$$\sqrt{\Delta} = \frac{5}{2}$$

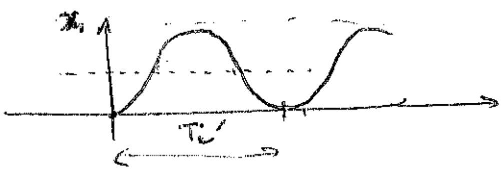
$$\begin{cases} \tilde{\omega}_1' = \dfrac{\frac{9}{2} - \frac{5}{2}}{7} = \dfrac{4/2}{7} = \dfrac{2}{7} \simeq 0{,}28 \\[2ex] \tilde{\omega}_2' = \dfrac{9/2 + 5/2}{7} = 1 \end{cases}$$

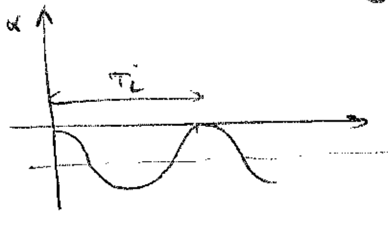
$\omega_i = \tilde{\omega}_i\left(\frac{\omega_0}{2}\right)$ $\to$ les pulsations baissent quasiment de moitiée.

l'arrivée de l'oiseau peut être considérée comme un échelon de force sur le système avec une masse.



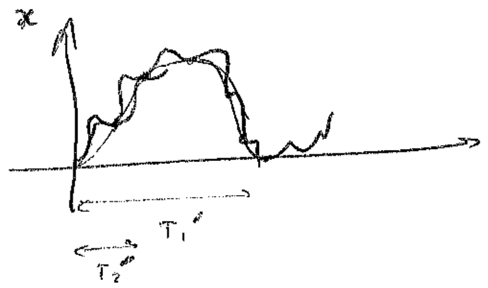
. ~~Sur~~ la réponse de chaque mode sera du type.

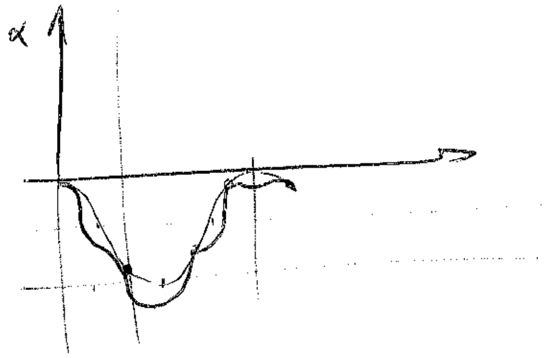




avec $T_i' = \frac{2\pi}{\omega_i'}$

d'où une réponse totale, superposition des deux réponses





la part de l'amplitude de chaque mode est calculable en décomposant la force sur les 2 vecteurs propres.

$$P = \vec{F} \cdot \vec{V_{G_2}}$$
$$= F\,H(t)\,\vec{i} \cdot \left[(\dot{x} - b\cos\alpha\,\dot{\alpha})\,\vec{i} + \cdot\,\vec{j} \right]$$
$$= \underbrace{F\,H(t)}_{Q_x}\,\dot{x} - \underbrace{b\,\cancel{}\,F\,H(t)}_{Q_\alpha}\,\dot{\alpha}$$

les équations de mouvement ont donc comme second membre.

$$\begin{matrix} \_\_\_\_ \\ \\ \_\_\_\_ \end{matrix} = \begin{bmatrix} F\,H(t) \\ \\ -b\,F\,H(t) \end{bmatrix}$$

Pour connaître la part (q_1, q_2) de chaque mode $\tilde{\omega}_1'$ et $\tilde{\omega}_2'$ il suffit de décomposer le chargement dans la base modale.

$$\begin{bmatrix} F\,H(t) \\ \\ -b\,F\,H(t) \end{bmatrix} = q_1 \begin{bmatrix} v_{11}' \\ \\ v_{12}' \end{bmatrix} + q_2 \begin{bmatrix} v_{21}' \\ \\ v_{22}' \end{bmatrix}$$