11h36 Examen Lpiau janvier 2012.

$h_1 = 0.3$ m

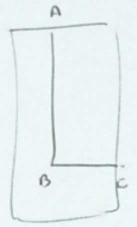
$\rho = 1000$ kg m^{-3}

$E = 2.1 \ 10^{11}$ Pa

$\nu = 0.28$

$\alpha = \frac{\pi}{4}$

1)



Bernoulli : ligne de courant A→C

↳ $P_A + \rho g z_A = P_C + \rho g z_C$

$P_C = P_A + \rho g (z_A - z_C)$

$P_C - P_A = \rho g h_1$

| ~~P_{atm}~~ P_a
| $\rho = 1000$ kg m^{-3}
| $g = 9.81$ m s^{-2}
| $h_1 = 3$

$P_C - P_A =$ ~~[illegible]~~ $0,294 \ 10^5$ Pa

11h42 2)

$F = P_C * S$

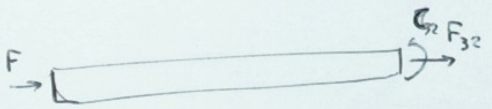
$= P_C \ \pi r_2^2$

$P_C = 0,294 \ 10^5$ Pa

$r_2 = 0,01$ m

$$F = 9,25 \text{ N}$$

11h46 3)



bilan des action

eau → $\{\tau_1\} = \begin{Bmatrix} -F\vec{x} \\ \vec{0} \end{Bmatrix}_C$

3 → $\{\tau_2\} = \begin{Bmatrix} \vec{F_{32}} \\ \vec{C_{32}} \end{Bmatrix}_D$

équilibre

→ $\Sigma \{\tau\} = \{0\}$

$$\left\{\begin{matrix} +F\vec{x} \\ \vec{0} \end{matrix}\right\}_C + \left\{\begin{matrix} \vec{F_{32}} \\ \vec{C_{32}} + \vec{F_{32}} \wedge \vec{DC} \end{matrix}\right\}_C = \{\vec{0}\}$$

$$\Rightarrow \quad \vec{F_{32}} = -F\vec{x}$$

$$\vec{C_{32}} + -F\vec{x} \wedge (-l_2\vec{x}) = 0$$

$$\hookrightarrow \vec{C_{32}} = \vec{0}$$

$$\{\mathcal{T}_{32}\} = \left\{\begin{matrix} -F\vec{x} \\ \vec{0} \end{matrix}\right\}_D = \left\{\begin{matrix} -9{,}25\ \vec{x}\ N \\ \vec{0} \end{matrix}\right\}_D$$

11h49 4)

$$\bar{\bar{\sigma}} = \begin{bmatrix} \sigma_{xx} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}_{(\ )\otimes(\ )}$$

Sur une surface, il faut que $\vec{T}(P,\vec{n}) = \bar{\bar{\sigma}}\,\vec{n}$

Pour la surface en C, normale $-\vec{x}$

$$\vec{T}(P,-\vec{x}) = \bar{\bar{\sigma}}\,(-\vec{x})$$

$$\frac{\vec{F}}{\pi r_2^2} = -\sigma_{xx}\,\vec{x}$$

$$\frac{F\vec{x}}{\pi r_2^2} = -\sigma_{xx}\,\vec{x}$$

$$\sigma_{xx} = -\frac{F}{\pi r_2^2}$$

$F = 9{,}25\ N$
$r_2 = 0.01\ m$

$$\boxed{\sigma_{xx} = -2{,}94\ 10^4\ Pa}$$

12h00 5)

$$\bar{\bar{\varepsilon}} = \frac{1+\nu}{E}\bar{\bar{\sigma}} - \frac{\nu}{E}(\text{trace}\ \bar{\bar{\sigma}})\,\bar{\bar{I_d}}$$

trace $\bar{\bar{\sigma}} = \sigma_{xx}$.

$$\bar{\bar{\varepsilon}} = \begin{bmatrix} \frac{\sigma_{xx}}{E} & 0 & 0 \\ 0 & -\frac{\nu}{E}\sigma_{xx} & 0 \\ 0 & 0 & -\frac{\nu}{E}\sigma_{xx} \end{bmatrix}_{(\ )\otimes(\ )}$$

comme $E = 2.1\ 10^{11}\ Pa$
$\nu = 0.28$

$$\bar{\bar{\varepsilon}} = 10^{-6}\begin{bmatrix} -0{,}14 & & \\ & +0{,}039 & \\ & & +0{,}039 \end{bmatrix}_{(\ )\otimes(\ )}$$

12h04 6) $\vec{v_C} = \vec{v_D} + \vec{\omega_D} \wedge \vec{DC} + \int_D^C \varepsilon_{xx} \, \vec{x} \, ds$

$\vec{v_D} = 0$
$\vec{\omega_D} = 0$) car encastrement

$\vec{v_C} = \varepsilon_{xx} \, \vec{x} \int_{x_D}^{x_C} (-dx)$

$= \varepsilon_{xx} \, \vec{x} \int_{x_C}^{x_D} dx$

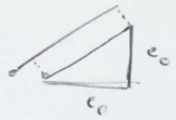
$= \varepsilon_{xx} \, \vec{x} \, [x]_{x_C}^{x_D}$

$= \varepsilon_{xx} \, l_2 \, \vec{x}$

$\varepsilon_{xx} = -0{,}14 \; 10^{-6}$
$l_2 = 5 \text{ m}$

$= -7{,}01 \; 10^{-7} \text{ m}$

12h05 7) Conservation du débit



$v_I S_I = v_J S_J$

$v_I (2\pi r_2)\left(\frac{U}{\sqrt{2}}\right) = v_J \left(2\pi (r_2 + e_0)\right)\left(\frac{U}{\sqrt{2}}\right)$

$\frac{v_J}{v_I} = \frac{r_2}{r_2 + e_0}$

$= \frac{r_2}{2 r_2}$

$= \frac{1}{2}$

12h12 8) Bernoulli entre A et J

$P_{atm} + \rho g z_A + \frac{\rho}{2} v_A^2 = P_{atm} + \rho g z_J + \frac{\rho}{2} v_J^2$

$\frac{\rho}{2} v_J^2 = \rho g (z_A - z_J)$

$V_J = \sqrt{2 g (h_1 - 2 r_2)} = 7{,}65 \text{ m s}^{-2}$

9) non proportionnel à h_1
~~non proportionnel à r_1~~ indépendant de r_1

12h18 10) $R_e = \frac{\rho V d}{\nu}$

en I $\quad R_{e_I} = \frac{\rho \, V_I \, \frac{U}{\sqrt{2}}}{\mu}$

$$[R_e] = \frac{kg\ m^{-3}\ m\ s^{-1}\ m}{Pa\ s}$$

$$= \frac{kg\ m^{-1}\ s^{-1}}{N\ m^{-2}\ s} = \frac{kg\ m^{-1}\ s^{-1}}{kg\ m\ s^{-2}\ m^{-2}\ s} = \frac{kg\ m^{-1}\ s^{-1}}{kg\ m^{-1}\ s^{-1}}$$

$= 1$ ok !

$\rho = 1000\ kg\ m^{-3}$

$V_I = V_J/2 = 3.82\ m s^{-1}$

$u = 7{,}01\ 10^{-7}\ m$

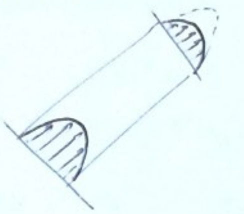
$\gamma = 10^{-3}\ Pa\ s$

$R_{e_J} = 1{,}89$

$R_{e_J} = \frac{R_{eI}}{2}$

$= 0{,}95$

12h24 11) l'écoulement est laminaire car $R_e < 2000$



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12h25 12) le rapport $\tilde{r}$ = longueur / gap est de

$$\tilde{r} = \frac{\overline{V_2}\ e_0}{\frac{u}{\overline{V_2}}}$$

$$= 2\ \frac{r_2}{u}$$

$r_2 = 0{,}01\ m$
$u = 7.01\ 10^{-7}\ m$

$= 2.85\ 10^4$

très élancé → pertes de charge régulières.

fin (12h28) → 52 minutes.