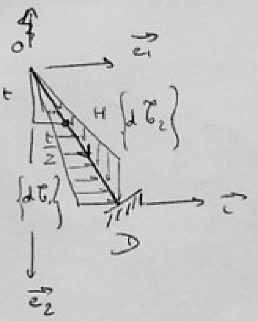
1)



2) pression

le problème est plan, épaisseur du barrage e dans la direction $\vec{k}$

. $\left\{ d\mathcal{T}_1 \right\} = \left\{ \begin{matrix} \omega g\, t_p e\, \vec{e_1}\, ds_p \\ 0\, \vec{k} \end{matrix} \right\}_P$

. $\left\{ d\mathcal{T}_2 \right\} = \left\{ \begin{matrix} \rho g e\, \tilde{h}(t_p)\, \vec{e_2}\, ds_p \\ 0\, \vec{k} \end{matrix} \right\}_P$

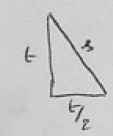
. encastrement en D

3) On oriente de O vers D

$$- \left\{ \mathcal{T}_{eff\,int} \right\} = \int_0^H \left\{ d\mathcal{T}_1 \right\} + \left\{ d\mathcal{T}_2 \right\}$$

Relations géométriques

*

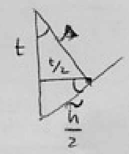


$$s^2 = t^2 + \frac{t^2}{4}$$

$$s^2 = t^2 \frac{5}{4}$$

$$s = \frac{\sqrt{5}}{2} t$$

*



$\frac{t}{2} \not\parallel t$

$$\frac{\frac{\tilde{h}}{2}}{t/2} = \frac{s}{t}$$

dans les triangles semblables

$$\text{d'où } \tilde{h} = \frac{2st}{2t}$$

$$\tilde{h} = s$$

$$- \left\{ \mathcal{T}_{eff\,int} \right\} = \int_0^H \omega g e \left\{ \begin{matrix} t_p\, \vec{e_1}\, ds_p \\ t_p\, \vec{e_1}\, ds_p \wedge \overrightarrow{PH} \end{matrix} \right\}_H + \rho e g \left\{ \begin{matrix} s_p\, \vec{e_2}\, ds_p \\ s_p \vec{e_2}\, ds_p \wedge \overrightarrow{PH} \end{matrix} \right\}_H$$

$$= \omega g e \left\{ \begin{matrix} \vec{e_1} \int_0^t t_p \frac{\sqrt{5}}{2}\, dt_p \\ \int_0^t \vec{e_1} \wedge \left[(t - t_p)\, \vec{e_2} + \left(\frac{t}{2} - \frac{t_p}{2} \right) \vec{e_1} \right] t_p \frac{\sqrt{5}}{2}\, dt_p \end{matrix} \right\}_H$$

$$+ \rho e g \left\{ \begin{matrix} \vec{e_2} \int_0^t \frac{\sqrt{5}}{2} t_p \frac{\sqrt{5}}{2}\, dt_p \\ \frac{\sqrt{5}}{2} \frac{\sqrt{5}}{2} \int_0^t t_p\, \vec{e_2}\, dt_p \wedge \left[(t - t_p)\, \vec{e_2} + \left(\frac{t}{2} - \frac{t_p}{2} \right) \vec{e_1} \right] \end{matrix} \right\}_H$$

$$= \omega g e \begin{Bmatrix} \dfrac{\sqrt{5}}{2} \dfrac{t^2}{2} \vec{e_1} \\ \dfrac{\sqrt{5}}{2} \vec{e_3} \displaystyle\int_0^t (t - t_p)\, t_p \, dt_p \end{Bmatrix}_H$$

$$+ \rho e g \begin{Bmatrix} \dfrac{5}{4} \dfrac{t^2}{2} \vec{e_2} \\ \dfrac{5}{4} (-\vec{e_3}) \displaystyle\int_0^t t_p \left(\dfrac{t}{2} - \dfrac{t_p}{2}\right) dt_p \end{Bmatrix}_H$$

$$= \omega g e \begin{Bmatrix} \dfrac{\sqrt{5}}{2} \dfrac{t^2}{2} \vec{e_1} \\ \dfrac{\sqrt{5}}{12} t^3 \vec{e_3} \end{Bmatrix}_H + \rho e g \begin{Bmatrix} \dfrac{5}{4} \dfrac{t^2}{2} \vec{e_2} \\ - \dfrac{t^3}{12} \vec{e_3} \end{Bmatrix}_H$$

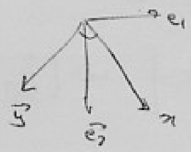
$$\{ \mathcal{T}_{eff\,int} \} = - \begin{Bmatrix} e t^2 \left[\omega g \dfrac{\sqrt{5}}{4} \vec{e_1} + \rho g \dfrac{5}{8} \vec{e_1} \right] \\ \dfrac{t^3}{12} e \left[\omega g \sqrt{5} - \rho g \right] \vec{e_3} \end{Bmatrix}_H$$

Composantes du torseur des effets intérieurs :

repère local $\quad \vec{x} = \dfrac{\frac{1}{2}\vec{e_1} + \vec{e_2}}{\sqrt{\frac{5}{4}}}$

$$\vec{x} = \frac{2}{\sqrt{5}} \left(\frac{1}{2} \vec{e_1} + \vec{e_2} \right)$$

$$\begin{cases} \vec{x} = \dfrac{1}{\sqrt{5}} \vec{e_1} + \dfrac{2}{\sqrt{5}} \vec{e_2} \\ \vec{y} = \dfrac{1}{\sqrt{5}} \vec{e_2} - \dfrac{2}{\sqrt{5}} \vec{e_1} \end{cases}$$



$$\cancel{\{ \mathcal{T}_{eff\,int} \} =} \quad 2\vec{x} + \vec{y} = \frac{4}{\sqrt{5}} \vec{e_2} + \frac{1}{\sqrt{5}} \vec{e_2}$$

$$2\vec{x} + \vec{y} = \sqrt{5}\, \vec{e_2}$$

$$\boxed{\vec{e_2} = \frac{2}{\sqrt{5}} \vec{x} + \frac{1}{\sqrt{5}} \vec{y}}$$

$$\vec{x} - 2\vec{y} = \frac{1}{\sqrt{5}} \vec{e_1} + \frac{4}{\sqrt{5}} \vec{e_1}$$

$$\boxed{\vec{e_1} = \frac{1}{\sqrt{5}} \vec{x} - \frac{2}{\sqrt{5}} \vec{y}}$$

$$\{ \mathcal{T}_{eff\,int} \} = - \begin{Bmatrix} \vec{x} \left[e t^2 \omega g \dfrac{1}{\sqrt{5}} \dfrac{\sqrt{5}}{4} + e t^2 \rho \dfrac{5}{8} \dfrac{2}{\sqrt{5}} g \right] + \vec{y} \left[e t^2 \omega g \dfrac{\sqrt{5}}{4} \left(-\dfrac{2}{\sqrt{5}} \right) + e t^2 \rho \dfrac{5}{8} \dfrac{1}{\sqrt{5}} g \right] \\ \dfrac{e t^3}{12} \left[\omega g \sqrt{5} - \rho g \right] \vec{e_3} \end{Bmatrix}$$

~~changement de base~~

$$\begin{cases} \text{l'effort normal est} & N = -\left[e t^2 \omega g \frac{1}{4} + e t^2 \rho g \frac{\sqrt{5}}{4}\right] \\ \text{l'effort tranchant est} & T_y = + e t^2 \omega g \frac{1}{2} - e t^2 \rho g \frac{\sqrt{5}}{8} \\ \text{le moment fléchissant est} & M_{f_3} = -\frac{e t^3}{12}\left[\omega g \sqrt{5} - \rho g\right] \end{cases}$$

Calcul du déplacement du p^t O.

$$\vec{U}_D = \vec{U}_O + \vec{\omega}_O \wedge \vec{OD} + \int_0^D \frac{N}{ES}\vec{x}\, ds + \int_0^D \left(\frac{M_{f_3}}{EI_{H_3}} \vec{3} \wedge \vec{PO}\right) ds$$

On néglige les déplacements dûs aux efforts tranchant devant ceux dûs aux moments fléchissant.

$$\vec{\omega}_D = \vec{\omega}_O + \int_0^D \frac{M_{f_3}}{EI_{H_3}} \vec{3}\, ds \qquad \text{avec } \vec{\omega}_D = \vec{0}$$

$$\vec{U}_O = \left[\int_0^D \frac{M_{f_3}}{EI_{H_3}} \vec{3}\, ds\right] \wedge \vec{OD}$$

$$+ \int_0^D \frac{eg\left[\frac{\omega}{4} + \rho\frac{\sqrt{5}}{4}\right] t^2}{E\, e\, \frac{\sqrt{5}}{2} t} \frac{\sqrt{5}}{2}\, dt\ \vec{x}$$

$$+ \int_0^D \frac{12\, eg}{12 E}\left[\frac{\omega\sqrt{5} - \rho}{e\left(\frac{\sqrt{5}}{2}t\right)^3}\right] t^3\ \frac{\sqrt{5}}{2} t\ \vec{3} \wedge (-\vec{x})\ \frac{\sqrt{5}}{2}\, dt$$

$$\vec{U}_O = -\oint \frac{\cancel{12}\cancel{e} g\ \cancel{8}^{\,2}}{\cancel{12} E \cancel{e}\ \cancel{8}\cancel{\sqrt{5}}\sqrt{5}} (\omega\sqrt{5} - \rho) \frac{\sqrt{5}}{\cancel{2}} \int_0^D dt\ \vec{3} \wedge \vec{x}\ \frac{\sqrt{5}}{\cancel{2}} h$$

$$+ \frac{\cancel{e} g\left[\frac{\omega}{4} + \rho\frac{\sqrt{5}}{4}\right]\sqrt{5}}{E\sqrt{5}\ \cancel{2}} \int_0^D t\, dt\ \vec{x}$$

$$+ \frac{g\ \cancel{8}^{\,2}\left[\omega\sqrt{5} - \rho\right]\sqrt{5}\sqrt{5}}{E\ \cancel{8}\sqrt{5}\ \cancel{2}\ \cancel{2}} \int_0^D t\, dt\ (-\vec{y})$$

$$\vec{U}_O = -\frac{g}{E}\frac{2h^2}{\sqrt{5}}\left[\omega\sqrt{5} - \rho\right]\vec{y}$$

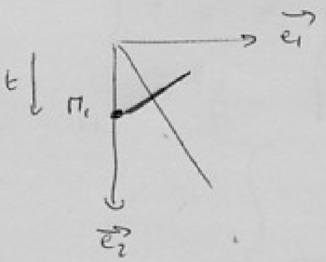
$$+ \frac{g}{E}\frac{h^2}{2}\left[\frac{\omega}{4} + \rho\frac{\sqrt{5}}{4}\right]\vec{x}$$

$$- \frac{g}{E}\frac{h^2}{\sqrt{5}}\left[\omega\sqrt{5} - \rho\right]\vec{y}$$

$$\vec{U_0} = -\frac{g}{E}\,\frac{3h^2}{\sqrt{5}}\left[\omega\sqrt{5}-\rho\right]\vec{y} + \frac{g}{E}\,\frac{h^2}{2}\left[\frac{\omega}{4}+\rho\frac{\sqrt{5}}{4}\right]\vec{x}$$

$$\vec{U_0} = \vec{e_1}\left[\frac{g}{E}\,\frac{h^2}{2}\left[\frac{\omega}{4}+\frac{\rho\sqrt{5}}{4}\right]\frac{1}{\sqrt{5}} + \frac{g}{E}\,\frac{3h^2}{\sqrt{5}}\left[\omega\sqrt{5}-\rho\right]\frac{2}{\sqrt{5}}\right]$$
$$+\ \vec{e_2}\left[\frac{g}{E}\,\frac{h^2}{2}\left[\frac{\omega}{4}+\frac{\rho\sqrt{5}}{4}\right]\frac{2}{\sqrt{5}} - \frac{g}{E}\,\frac{3h^2}{\sqrt{5}}\left[\omega\sqrt{5}-\rho\right]\frac{1}{\sqrt{5}}\right]$$

5)



$$\sigma = \frac{N}{S} - \frac{Mf_3}{I h_3}\,\frac{h}{2}$$

$$\sigma = \frac{\cancel{2}\,\cancel{E}\,g\,t^{\cancel{2}}}{\cancel{4}\,\underset{2}{\cancel{4}}\,\cancel{E}\sqrt{5}}\left[-\omega-\rho\sqrt{5}\right] + \frac{\cancel{12}\,\cancel{E}\,g\,\cancel{t^3}}{\cancel{12}\,\cancel{E}\left(\frac{\sqrt{5}}{\cancel{2}}t\right)^{\cancel{3}}}\left[\omega\sqrt{5}-\rho\right]\frac{\sqrt{5}}{\cancel{4}}\,t^2$$

$$\sigma = g t\left[\frac{2}{5}\left[\omega\sqrt{5}-\rho\right] + \frac{1}{2\sqrt{5}}\left[-\omega-\rho\sqrt{5}\right]\right]$$

$$= g t\left[\omega\left(\frac{2}{\sqrt{5}}-\frac{1}{2\sqrt{5}}\right) - \rho\left(\frac{2}{5}+\frac{1}{2}\right)\right]$$

$$\sigma = g t\left[\frac{\omega}{\sqrt{5}}\,\frac{3}{2} - \frac{9}{10}\rho\right]$$