EXO1 :

TD1

1°/ Torseurs correspondant à chaque action :

(1)

$$\left\{ \text{rotor p}^{al} \rightarrow S \right\} = \left\{ \begin{matrix} \vec{F} \\ \vec{M} \end{matrix} \right\}_A = \left\{ \begin{matrix} \vec{F} \\ \vec{M} + \vec{F} \wedge \vec{AG} \end{matrix} \right\}_G$$

$$\left\{ \text{rotor a-c} \rightarrow S \right\} = \left\{ \begin{matrix} \vec{Q} \\ \vec{M_Q} \end{matrix} \right\}_B = \left\{ \begin{matrix} \vec{Q} \\ \vec{M_Q} + \vec{Q} \wedge \vec{BG} \end{matrix} \right\}_G$$

$$\left\{ \text{pesanteur} \rightarrow S \right\} = \left\{ \begin{matrix} m\vec{g} \\ \vec{0} \end{matrix} \right\}_G$$

$$\left\{ \text{resistance air} \rightarrow S \right\} = \left\{ \begin{matrix} \vec{R} \\ \vec{0} \end{matrix} \right\}_C = \left\{ \begin{matrix} \vec{R} \\ \vec{R} \wedge \vec{CG} \end{matrix} \right\}_G$$

2°/ PFS $\Rightarrow \left\{ \begin{matrix} \vec{F} \\ \vec{M_G} \end{matrix} \right\} = \left\{ \begin{matrix} \vec{0} \\ \vec{0} \end{matrix} \right\}$

Soit
$$\begin{cases} \vec{F} + \vec{Q} + m\vec{g} + \vec{R} = \vec{0} \quad \dots (1) \\ \vec{M} + \vec{F} \wedge \vec{AG} + \vec{M_Q} + \vec{Q} \wedge \vec{BG} + \vec{R} \wedge \vec{CG} = \vec{0} \quad \dots (2) \end{cases}$$

$$(1) \Rightarrow \begin{cases} F_x + R = 0 \\ F_y + Q = 0 \\ F_z - mg = 0 \Rightarrow F_z = 3\,10^4 N \end{cases}$$

$$(2) \Rightarrow \begin{vmatrix} 0 \\ 0 \\ M \end{vmatrix} + \begin{vmatrix} F_x \\ F_y \\ F_z \end{vmatrix} \wedge \begin{vmatrix} AG_x \\ 0 \\ AG_z \end{vmatrix} + \begin{vmatrix} 0 \\ M_Q \\ 0 \end{vmatrix} + \begin{vmatrix} 0 \\ Q \\ 0 \end{vmatrix} \wedge \begin{vmatrix} BG_x \\ 0 \\ BG_z \end{vmatrix} + \begin{vmatrix} R \\ 0 \\ 0 \end{vmatrix} \wedge \begin{vmatrix} CG_x \\ 0 \\ 0 \end{vmatrix} = \begin{vmatrix} 0 \\ 0 \\ 0 \end{vmatrix}$$

EXO1

TD1

$F_Y\, AG_z + Q\, BG_z = 0 \iff F_Y(AG_z - BG_z) = 0$

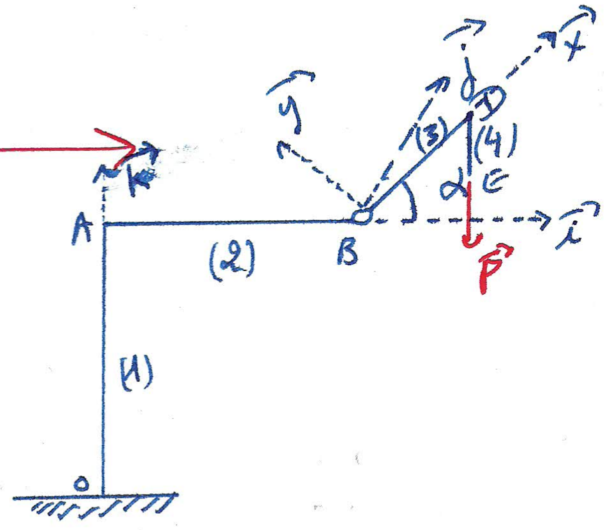
$F_z\, AG_x - F_x\, AG_z + M_Q = 0 \iff F_x = \dfrac{F_z\, AG_x + M_Q}{AG_z} = \dfrac{3 \cdot 10^4(-0,2) - 3 \cdot 2}{-1,5}$

$\Rightarrow \boxed{F_x = \cancel{}\ 4002\,N}$

$M - F_Y\, AG_x - Q\, BG_x = 0 \Rightarrow M - F_Y(AG_x + BG_x) = 0$

$\Rightarrow F_Y = \dfrac{M}{AG_x + BG_x} = \dfrac{400}{-0,2-4} = -95,2\,N$

$\Rightarrow \boxed{Q = 95\,N}$



EXO2 : isolement d'un système

act° en D :

$\uparrow \vec{R_D}$, $\downarrow \vec{P}$

$\{P \to 4\} = \begin{Bmatrix} \vec{P} \\ \vec{0} \end{Bmatrix}_E$

$\{3 \to 4\} = \begin{Bmatrix} \vec{R_D} \\ \vec{0} \end{Bmatrix}_D = \begin{Bmatrix} \vec{R_D} \\ \underbrace{\vec{R_D} \wedge \vec{DE}}_{\vec{0}} \end{Bmatrix}_E$

PFS $\begin{cases} \vec{P} + \vec{R_D} = 0 \Rightarrow \vec{R_D} = -\vec{P} \Rightarrow \boxed{R_D = 10\,kN} \\ \vec{0} = \vec{0} \end{cases}$

act° en B :

$\{4 \to 3\} = \begin{Bmatrix} \vec{P} \\ \vec{0} \end{Bmatrix}_D = \begin{Bmatrix} \vec{P} \\ \vec{P} \wedge \vec{DB} \end{Bmatrix}_B$

$\{pes \to 3\} = \begin{Bmatrix} m_3 \vec{g} \\ \vec{0} \end{Bmatrix}_{G_3} = \begin{Bmatrix} m_3 \vec{g} \\ m_3 \vec{g} \wedge \vec{G_3B} \end{Bmatrix}_B$

$\{2 \to 3\} = \begin{Bmatrix} \vec{R_B} \\ \vec{M_B} \end{Bmatrix}_B$

Exo2 :

TD1

3

PFS : $\begin{cases} \vec{P} + m_3\vec{g} + \vec{R_B} = \vec{0} & (1) \\ \vec{P} \wedge \vec{DB} + m_3\vec{g} \wedge \vec{G_3B} + \vec{M_B} = \vec{0} & (2) \end{cases}$; $\vec{DB} = -DB\,\vec{x}$

(1) $\begin{cases} 0 + 0 + R_{Bx} = 0 \\ 0 + 0 + R_{By} = 0 \\ -P - m_3 g + R_{Bz} = 0 \end{cases} \Rightarrow R_{Bz} = P + m_3 g = 10^4 + 50\times10 = 10\,500\,N$

(2) $\begin{vmatrix} 0 \\ 0 \\ -P \end{vmatrix} \wedge \begin{vmatrix} -\cos\alpha\,|DB| \\ -\sin\alpha\,|DB| \\ 0 \end{vmatrix} + \begin{vmatrix} 0 \\ 0 \\ -m_3 g \end{vmatrix} \wedge \begin{vmatrix} -\frac{1}{2}\cos\alpha\,|DB| \\ -1/2\sin\alpha\,|DB| \\ 0 \end{vmatrix} + \begin{vmatrix} M_{Bx} \\ M_{By} \\ M_{Bz} \end{vmatrix} = \begin{vmatrix} 0 \\ 0 \\ 0 \end{vmatrix}$

$\begin{cases} -P\sin\alpha\,|DB| - m_3 g\,\frac{1}{2}\sin\alpha\,|DB| + M_{Bx} = 0 \\ P\cos\alpha\,|DB| + m_3 g\,\frac{1}{2}\cos\alpha\,|DB| + M_{By} = 0 \\ M_{Bz} = 0 \end{cases}$

$\begin{cases} M_{Bx} = \left[P + \left(\frac{m_3 g}{2}\right)\right]\sin\alpha\,|DB| \\ M_{By} = -\left(P + \frac{m_3}{2} g\right)\cos\alpha\,|DB| \\ M_{Bz} = 0 \end{cases}$

act° en A :

$\{1 \to 2\} = \begin{Bmatrix} \vec{R_A} \\ \vec{M_A} \end{Bmatrix}_A$; $\{pes \to 2\} = \begin{Bmatrix} m_2\vec{g} \\ \vec{0} \end{Bmatrix}_{G_2} = \begin{Bmatrix} m_2\vec{g} \\ m_2\vec{g} \wedge \vec{G_2A} \end{Bmatrix}_A$

$\{3 \to 2\} = \begin{Bmatrix} -\vec{R_B} \\ -\vec{M_B} \end{Bmatrix}_B = \begin{Bmatrix} -\vec{R_B} \\ -\vec{M_B} - \vec{R_B} \wedge \vec{BA} \end{Bmatrix}_A$

PFS : $\begin{cases} \vec{R_A} + m_2\vec{g} + \vec{R_B} = 0 \\ \vec{M_A} + m_2\vec{g} \wedge \vec{G_2A} - \vec{M_B} - \vec{R_B} \wedge \vec{BA} = \vec{0} \end{cases}$

$\circledast$ $R_{Ax} = 0$; $R_{Ay} = 0$; $R_{Az} = -m_2 g - R_{Bz} = 0 \Rightarrow R_{Az} = m_2 g + R_{Bz}$

$R_{Az} = 800 + 10500 = 11300\,N$

$\circledast$ $\begin{cases} M_{Ax} - M_{Ay} = 0 \Rightarrow M_{Ax} = M_{Bx} \\ M_{Ay} - m_2 g \cdot G_2A_x - M_{By} - R_{Bz} \cdot BA_x = 0 \\ M_{Az} = 0 \end{cases}$

Exo 2 :

act° en O :

$$\{2\to 1\} = \begin{Bmatrix} -\vec{R_A} \\ -\vec{M_A} \end{Bmatrix}_A = \begin{Bmatrix} -\vec{R_A} \\ -M_A - \vec{R_A} \wedge \vec{AO} \end{Bmatrix}_O$$

$$\{pes \to 1\} = \begin{Bmatrix} m_1\vec{g} \\ \vec{0} \end{Bmatrix}_{G_1} = \begin{Bmatrix} m_1\vec{g} \\ m_1\vec{g} \wedge \vec{G_1O} \end{Bmatrix}_O$$

$$\{0\to 1\} = \begin{Bmatrix} \vec{R_0} \\ \vec{M_0} \end{Bmatrix}_0$$

PFS : $\begin{cases} R_{0x} = 0 \\ R_{0y} = 0 \\ R_{0z} = R_{Az} + m_1\vec{g} \end{cases}$; $\begin{cases} M_{0x} = M_{Ax} = M_{Bx} \\ M_{0y} - M_{Ay} = 0 \Rightarrow M_{0y} = M_{Ay} \\ M_{0z} - M_{Az} = 0 \Rightarrow M_{0z} = M_{Az} = 0 \end{cases}$

2°/ AN :

Action en B : $\boxed{R_{Bz} = 10500 N}$

$$M_{Bx}(maxi) = +\left(P + \frac{m_3 g}{2}\right)|DB| = \left(10^4 + \frac{50}{2}10\right)2 = \begin{pmatrix} 20500 N.m \\ \alpha = 90° \end{pmatrix}$$

$$= \left(\begin{matrix} -20500 N.m \\ \alpha = -90 \end{matrix}\right)$$

$$M_{By}(maxi) = -\left(P + \frac{m_3 g}{2}\right)|DB|$$
$$= -20500 Nm \ (\alpha = 0)$$

Action en A : $\boxed{R_{Az} = 11300 N}$

$$M_{Ax}(max) = M_{Bx}(max) = \pm 20500 N.m$$

$$M_{Ay} = m_2 g \ G_2A_x + M_{By} + R_{Bz} \ BA_x$$
$$= 80.10\left(-\frac{1,5}{2}\right) - 20500 + 10500 \ (-1,5)$$
$$= 36850 N.m$$

Act° en O : $R_{0z} = R_{Az} + m_1 g = 11300 + 120.10 = 12500 N$

$$\begin{cases} M_{0x\,max} = -20500 Nm \\ M_{0y} = 36850 N.m \\ M_{0z} = 0 \end{cases}$$

:) "THE END"