



les deux paramètres du mouvement sont :

$$T = T_1 + T_2 + T_3$$

$$T_1 = \frac{1}{2} m_1 \dot{y}_1^2$$

$$T_2: \vec{OG}_2 = \vec{OA} + \vec{AB} + \vec{BG}_2$$

$$= y_1 \vec{z}_0 + l_1 \vec{y}_0 + \frac{l_2}{2} \vec{v}$$

$$\frac{d\vec{OG}_2}{dt} = \dot{y}_1 \vec{z}_0 + \frac{l_2}{2} \dot{\theta} \vec{v}$$

$$= \left[ \dot{y}_1 + \frac{l_2}{2} (-\sin\theta) \dot{\theta} \right] \vec{z}_0 + \left[ \frac{l_2}{2} \dot{\theta} \cos\theta \right] \vec{z}_0$$

$$T_2 = \frac{1}{2} \left[ \left[ \dot{y}_1 + \frac{l_2}{2} \dot{\theta} \sin\theta \right]^2 + \left[ \frac{l_2}{2} \dot{\theta} \cos\theta \right]^2 \right] m_2$$

$$+ \frac{1}{2} \begin{bmatrix} \dot{\theta} & 0 & 0 \end{bmatrix} \begin{matrix} A \\ C \end{matrix} \begin{bmatrix} \dot{\theta} \\ 0 \\ 0 \end{bmatrix} \begin{matrix} (x) \\ (y, z, z_0) \end{matrix}$$

$$\text{avec } A = \int_{-l_2/2}^{l_2/2} v^2 dv_p \quad \text{avec } \rho = \frac{m_2}{l_2}$$

$$= \rho \frac{l_2^3}{3 \cdot 4}$$

$$= m_2 \frac{l_2^2}{3 \cdot 4}$$

$$= \frac{m_2 l_2^2}{12}$$

$$T_2 = \frac{1}{2} \left[ \dot{y}_1^2 + \frac{l_2^2}{4} \dot{\theta}^2 - \dot{y}_1 \dot{\theta} l_2 \sin\theta \right] m_2$$

$$+ \frac{1}{2} \frac{m_2 l_2^2}{12} \dot{\theta}^2$$

$$= \frac{1}{2} \dot{y}_1^2 m_2 + \frac{1}{2} \frac{m_2 l_2^2}{3} \dot{\theta}^2 - \frac{1}{2} \dot{y}_1 \dot{\theta} m_2 l_2 \sin\theta$$

$$T_3 = \frac{1}{2} m \left[ \dot{y}_1^2 + l_2^2 \dot{\theta}^2 - 2 \dot{y}_1 \dot{\theta} l_2 \sin\theta \right]$$

$$2T = \dot{y}_1^2 [m_1 + m_2 + \pi] + \dot{y}_1 \dot{\theta} [-m_2 l_2 \sin \theta - 2\pi l_2 \sin \theta] + \dot{\theta}^2 \left[ \frac{m_2 l_2^2}{12} + \pi l_2^2 \right]$$

$$\frac{\partial T}{\partial \dot{y}_1} = [m_1 + m_2 + \pi] \dot{y}_1 + [-l_2 \sin \theta (m_2 + 2\pi)] \dot{\theta}$$

$$\frac{d}{dt} \frac{\partial T}{\partial \dot{y}_1} = [m_1 + m_2 + \pi] \ddot{y}_1 + [$$

$$- \frac{\partial T}{\partial y_1} = 0$$

$$\frac{\partial T}{\partial \dot{\theta}} = \left[ \frac{m_2 l_2^2}{12} + \pi l_2^2 \right] \dot{\theta} + [-l_2 \sin \theta (m_2 + 2\pi)] \dot{y}_1$$

$$\frac{d}{dt} \frac{\partial T}{\partial \dot{\theta}} = [ \ddot{\theta} + [ -l_2 \cos(\theta) (m_2 + \pi) \dot{y}_1 \dot{\theta} ]$$

$$- \frac{\partial T}{\partial \theta} = l_2 (m_2 + 2\pi) \dot{y}_1 \dot{\theta} \cos \theta$$

Puissance des efforts extérieurs.

permanente sur chaque solide + force extérieure sur  $D_1$

$$P = \vec{F} \cdot \vec{V}_{G_1, \text{CS}1/R_0} + m_1 g (-\vec{z}_0) \cdot \vec{V}_{G_1, \text{CS}1/R_0} + m_2 g (-\vec{z}_0) \cdot \vec{V}_{G_2, \text{CS}2/R_0} + \pi g (-\vec{z}_0) \cdot \vec{V}_{C, \text{CS}2/R_0}$$

$$= F_0 \cos \omega t \dot{y}_1 + 0$$

$$+ (-m_2 g) \cdot \left( \frac{l}{2} \dot{\theta} \cos \theta \right)$$

$$+ (-\pi g) \cdot (l \dot{\theta} \cos \theta)$$

$$G. P = Q_{y_1} \dot{y}_1 + Q_{\theta} \dot{\theta}$$

$$\text{d'où } \begin{cases} Q_{y_1} = F \cos \omega t \\ Q_{\theta} = \left( -\frac{m_2}{2} - \pi \right) g l \cos \theta \end{cases}$$

Equations du mouvement :

$$\frac{d}{dt} \frac{\partial T}{\partial \dot{q}_i} - \frac{\partial T}{\partial q_i} = Q_i$$

en  $S_1$  :

$$-1/2 (m_2 + M) l_2^2 \theta^2 \cos(\theta)$$

$$\left[ m_1 + m_2 + M \right] \ddot{y}_1 + \left[ -\frac{l_2}{2} \sin \theta (m_2 + 2M) \right] \ddot{\theta} = F \cos \theta$$

en  $\theta$  :

$$\left[ \frac{m_2 l_2^2}{3} + M l_2^2 \right] \ddot{\theta} + \left[ -l_2 \sin \theta (m_2 + 2M) \right] \ddot{y}_1 = - \left( \frac{m_2}{2} + M \right) g l \cos \theta$$

corection mise au point par Sylvain Mézil et Jean-Michel Génevaux