

Exo 1

T.D. n° 6

① paramétrage : (S1) → translation : $\vec{OA} = y \vec{y}_0$
 (S2) → rotation : $\theta (\vec{m})$

①

② Energie cinétique :

$$2 T(\Sigma/R_0) = m_1 \vec{v}_0^2(G_1) + m_2 \vec{v}_0^2(G_2) + \left(\vec{\Omega}_{02} \cdot \underset{\times \vec{\Omega}_{02}}{J}(G_2, S_2) \right) + M \vec{v}_0^2(C)$$

$$2 T(\Sigma/R_0) = m_1 \dot{y}_1^2 + m_2 \left(\dot{y} \vec{y}_0 + \frac{l_2}{2} \dot{\theta} \vec{z}_2 \right)^2 + m_2 \frac{l_2^2}{3} \dot{\theta}^2 + M \left(\dot{y} \vec{y}_0 + l_2 \dot{\theta} \vec{z}_2 \right)^2$$

$$2 T(\Sigma/R_0) = (m_1 + m_2 + M) \dot{y}^2 + \dot{\theta}^2 \left(M l_2^2 + m_2 \frac{l_2^2}{3} \right) - (m_2 + 2M) \dot{y} l_2 \dot{\theta} \sin \theta$$

$$2 T(\Sigma/R_0) = M_T \dot{y}^2 + I \dot{\theta}^2 - (m_2 + 2M) \dot{y} l_2 \dot{\theta} \sin \theta$$

avec : $M_T = m_1 + m_2 + M$; Masse totale du système

$I = M l_2^2 + m_2 \frac{l_2^2}{3}$; Moment d'inertie par rapport (B, \vec{x}_0) du "bras 2 + pince"

③ Energie potentielle :

$$E_p(\Sigma) = \left(\frac{m_2}{2} + M \right) g l_2 \sin \theta + C^E$$

L'équation de Lagrange par rapport à une variable généralisée (q_i): (2)

$$\left(\frac{d}{dt} \frac{\partial}{\partial \dot{q}_i} - \frac{\partial}{\partial q_i} \right) T(S/R_0) = \left\{ \bar{S} \rightarrow S \right\} \times \left\{ V_{q_i}(S/R_0) \right\}$$

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$$\bullet \frac{\partial T}{\partial \theta} = - \frac{(m_2 + 2M) \dot{y} l_2 \dot{\theta} \cos \theta}{2}$$

$$\bullet \frac{\partial T}{\partial y} = 0$$

$$\bullet \frac{\partial T}{\partial \dot{\theta}} = \frac{I}{2} 2 \dot{\theta} - (m_2 + 2M) \dot{y} \frac{l_2}{2} \sin \theta$$

$$\bullet \frac{\partial T}{\partial \dot{y}} = M_T \dot{y} - \frac{(m_2 + 2M)}{2} l_2 \dot{\theta} \sin \theta$$

Lagrange (θ): $\left(\frac{d}{dt} \frac{\partial}{\partial \dot{\theta}} - \frac{\partial}{\partial \theta} \right) T(S/R_0) = \left\{ \bar{S} \rightarrow S \right\} \times \left\{ V_{\theta}(S/R_0) \right\} + \Gamma$

$$I \ddot{\theta} - (m_2 + 2M) \frac{l_2}{2} \left(\ddot{y} \sin \theta + \dot{y} \cos \theta \dot{\theta} \right) - \frac{(m_2 + 2M)}{2} \dot{y} l_2 \dot{\theta} \cos \theta$$

$$= I \ddot{\theta} - \frac{(m_2 + 2M)}{2} l_2 \ddot{y} \sin \theta = \underbrace{\left\{ \bar{S} \rightarrow S \right\} \times \left\{ V_{\theta}(S/R_0) \right\}}_{(2)} + \Gamma$$

cherchons le terme @:

$$\text{pes} \rightarrow (S_2) \Rightarrow \left\{ \begin{array}{l} -m_2 g \vec{z}_0 \\ -\frac{l_2}{2} m_2 g \vec{x}_2 \end{array} \right\}; \quad V_{\theta, m_2} = \frac{\partial}{\partial \dot{\theta}} \left\{ \begin{array}{l} \dot{\theta} \vec{x}_2 \\ \frac{l_2}{2} \dot{\theta} z_2 \end{array} \right\}$$

$$\text{pes} \rightarrow (P) \Rightarrow \left\{ \begin{array}{l} -Mg \vec{z}_0 \\ -l_2 Mg \vec{x}_2 \end{array} \right\}; \quad V_{\theta, M} = \frac{\partial}{\partial \dot{\theta}} \left\{ \begin{array}{l} \dot{\theta} \vec{x}_2 \\ l_2 \dot{\theta} z_2 \end{array} \right\}$$

$$\left\{ \bar{S} \rightarrow S \right\} \times \left\{ V_{\theta}(S/R_0) \right\}$$

$$= - \frac{l_2}{2} m_2 g - \frac{l_2}{2} m_2 g \cos \theta - Mg l_2 \cos \theta - Mg l_2$$

$$= - l_2 g \left(M + \frac{m_2}{2} \right) - g l_2 \cos \theta \left(\frac{m_2}{2} + M \right)$$

D'où la 1^{ère} équation de Lagrange en θ :

(3)

$$I \ddot{\theta} - \frac{(M_2 + 2M)}{2} l_2 \ddot{y} \sin \theta = - l_2 g (M + \frac{m_2}{2}) - g l_2 \cos \theta (\frac{m_2}{2} + M) + \Gamma$$

$$\Rightarrow \Gamma = I \ddot{\theta} - \frac{(m_2 + 2M)}{2} l_2 \ddot{y} \sin \theta + (\frac{M + m_2}{2}) g l_2 \cos \theta + (\frac{2M + m_2}{2}) l_2 g$$

Lagrange (y):

$$\{ \text{Pes} \rightarrow S_1 \} = \left\{ \begin{array}{l} -m_1 g \vec{z}_0 \\ \vec{0} \end{array} \right\}; \{ \text{sol} \rightarrow S_1 \} = \left\{ \begin{array}{l} \vec{R} \\ \vec{0} \end{array} \right\}$$

$$V_{y_{m_1}} = \frac{\partial}{\partial y} \left\{ \begin{array}{l} \vec{0} \\ y \vec{y}_0 \end{array} \right\}; V_{y_{m_2}} = \frac{\partial}{\partial y} \left\{ \begin{array}{l} \vec{0} \\ y \vec{y}_0 \end{array} \right\}$$

\Rightarrow En suivant la même démarche:

$$F = M_T \ddot{y} - \left(\frac{M_2 + 2M}{2} \right) l_2 \left(\ddot{\theta} \sin \theta + \dot{\theta}^2 \cos \theta \right)$$

