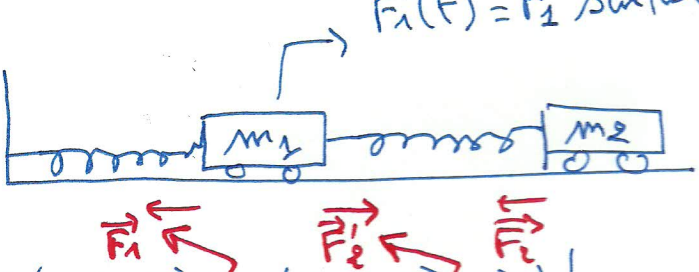
TD n°4

Exo1

TGD : $\{D(S/R_0)\} = \{\overline{S} \to S\}$

1°/ Définir référentiel : Centre de gravité



2°/ solide ①

$$\{\text{ressort}_2 \to S_1\} = \left\{ \begin{matrix} \vec{F_1}\sin(\omega t) + (-k_1\vec{x_1}) + (k_2(\vec{x_2} - \vec{x_1})) \\ \vec{0} \end{matrix} \right\}_{G_1}$$

solide ②

$$\{\text{ressort} \to S_2\} = \left\{ \begin{matrix} -k_2(\vec{x_2} - \vec{x_1}) \\ \vec{0} \end{matrix} \right\}_{G_2}$$

3°/ TGD : Torseur dynamique

solide ① : $\{D(S_1/R_0)\}_{G_1} = \left\{ \begin{matrix} m_1 \ddot{\vec{x}}_1 \\ \vec{0} \end{matrix} \right\}$

solide ② : $\{D(S_2/R_0)\}_{G_2} = \left\{ \begin{matrix} m_2 \ddot{\vec{x}}_2 \\ \vec{0} \end{matrix} \right\}$

4°/ $\begin{cases} m_1 \ddot{x}_1 = F_1 \sin(\omega t) - k_1 x_1 + k_2 x_2 - k_2 x_1 = -x_1(k_1 + k_2) + k_2 x_2 \\ m_2 \ddot{x}_2 = k_2 x_1 - k_2 x_2 \end{cases}$

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} F_1 \sin \omega t \\ 0 \end{bmatrix}$$

Exo 2

- $\{$ Viscosité $\longrightarrow (S)\}$

$$\left\{\begin{array}{l} -(c_1+c_2)\dot{x} + c_1 l_1 \dot{\theta} - l_2 c_2 \dot{\theta} \\ -l_2 c_2 \dot{x} + l_1 c_1 \dot{x} - \dot{\theta} l_1^2 c_1 - \dot{\theta} l_2^2 c_2 \end{array}\right\}_G$$

- $\{$ élasticité (ressort) $\longrightarrow (S)\}$

$$\left\{\begin{array}{l} k_1(l_0 - (x - l_1\theta)) + k_2(l_0 - (x - l_2\theta)) \\ l_0(l_1 k_1 + l_2 k_2) - x(l_1 k_1 - l_2 k_2) + (l_1^2 k_1 + l_2^2 k_2)\theta \end{array}\right\}$$

Poids : $\left\{\begin{array}{l} -mg \\ 0 \end{array}\right\}_G$; Fcs ext $\left\{\begin{array}{l} F_x(t) \\ M_{Gx}(t) \end{array}\right\}$

$$\{D\} = \left\{\begin{array}{l} m\ddot{x} \\ J\ddot{\theta} \end{array}\right\}$$

D'où :

$$\begin{bmatrix} m & 0 \\ 0 & J \end{bmatrix} \left\{\begin{array}{l} \ddot{x} \\ \ddot{\theta} \end{array}\right\} + \begin{bmatrix} c_1+c_2 & -l_1 c_1 + l_2 c_2 \\ -l_1 c_1 + l_2 c_2 & l_1^2 c_1 + l_2^2 c_2 \end{bmatrix} \left\{\begin{array}{l} \dot{x} \\ \dot{\theta} \end{array}\right\}$$

$$+ \begin{bmatrix} k_1+k_2 & -l_1 k_1 + l_2 k_2 \\ -l_1 k_1 + l_2 k_2 & l_1^2 k_1 + l_2^2 k_2 \end{bmatrix} \begin{bmatrix} x \\ 0 \end{bmatrix}$$

$$= \left\{\begin{array}{l} F_x(t) + l_0(k_1+k_2) - mg \\ M_{Gx}(t) + l_0(l_1 k_1 + l_2 k_2) \end{array}\right\}$$