TD5 : Dynamique - Energie Cinétique.

TEC - Ensemble de solides - liaison parfaite

On isole le syst $\Sigma = \{1, 2, 3\}$ (1)

$R_0 = (O, x_0 y_0 z_0)$ réf. galiléen

T.E.C de Σ dans R_0 est :

$$\frac{d}{dt}\left[T(\Sigma/R_0)\right] = P_{(\bar{\Sigma} \to \Sigma/R_0)} + P_{\text{intereffort}}$$

Liaisons Parfaites $\Rightarrow$ $P_{\text{intereffort}} = 0$.

__calcul des Puissances :__

$$P(\bar{\Sigma} \to \Sigma/R_0) = P_{pes \to 1} + P_{pes \to 2} + P_{pes \to 3} + P_{lame \to 3}$$
$$+ P_{moteur \to 1}$$

- $$P_{pes \to 1} = \begin{Bmatrix} -m_1 g \vec{z_0} \\ \vec{0} \end{Bmatrix}_{G_1} \times \begin{Bmatrix} \omega_0 \vec{x_0} \\ \vec{V}_{G_1 \in 1/R_0} = \vec{0} \end{Bmatrix}_{G_1}$$

car G_1 est fixe

donc : $\boxed{P_{pes \to 1} = 0}$

- $$P_{pes \to 2} = \begin{Bmatrix} -m_2 g \cdot \vec{z_0} \\ \vec{0} \end{Bmatrix}_{G_2} \times \begin{Bmatrix} \vec{\Omega}_{2/R_0} = \vec{0} \\ \vec{V}(G_2 \in 2/R_0) \end{Bmatrix}_{G_2}$$

$$\vec{V}(G \in 2/R_0) = \frac{d}{dt}\vec{OG_2}\Big|_{R_0} = \frac{d}{dt}\left[\vec{OK} + \vec{KG_2}\right]_{R_0}$$

$$\vec{V}(G_2 \in 2/R_0) = \frac{d}{dt}\left[-a\,\vec{x_3} + \lambda(t)\,\vec{z_0}\right]_{R_0}$$ ②

$$= \dot{\lambda}\,\vec{z_0} + \cancel{\frac{d}{dt}\left(-a\vec{x_3}\right)_{R_3}}^{\,0} + \vec{\Omega}_{R_3/R_0} \wedge \left(-a\vec{x_3}\right)$$

$$= \dot{\lambda}(t)\,\vec{z_0} + \left(\dot{\beta}\cdot\vec{z_0}\right) \wedge \left(-a\,\vec{x_3}\right)$$

$$\Rightarrow \vec{V}(G_2 \in 2/R_0) = \dot{\lambda}(t)\cdot\vec{z_0} - a\,\dot{\beta}\,\vec{y_3}$$

Donc : $\boxed{P_{pes \to 2} = -m_2\, g\, \dot{\lambda}(t)}$

- $P_{pes \to 3} = \left\{\begin{matrix} -m_3 g\,\vec{z_0} \\ \vec{0} \end{matrix}\right\}_{G_3=0} \times \left\{\begin{matrix} \dot{\beta}\cdot\vec{z_0} \\ \vec{0} \end{matrix}\right\}_{G_3=0}$

$$\boxed{P_{pes \to 3} = 0}$$

- $P_{lame \to 3} = \left\{\begin{matrix} F\cdot\vec{y_0} \\ \vec{0} \end{matrix}\right\}_P \times \left\{\begin{matrix} \vec{\Omega}_{3/R_0} = \dot{\beta}\,\vec{z_0} \\ \vec{V}(P \in 3/R_0) \end{matrix}\right\}_P$

$$\vec{V}(P \in 3/R_0) = \frac{d}{dt}\left(\vec{OP}\right)_{R_0} = \frac{d}{dt}\left(b\,\vec{x_3} - c\,\vec{z_0}\right)_{R_0}$$

$$= \cancel{\frac{d}{dt}\left(b\vec{x_3}\right)_{R_3}}^{\,0} + \vec{\Omega}_{R_3/R_0} \wedge b\,\vec{x_3}$$

$$= \dot{\beta}\,\vec{z_0} \wedge b\,\vec{x_3} = b\cdot\dot{\beta}\,\vec{y_3}$$

$$\boxed{\vec{V}(P \in 3/R_0) = b\,\dot{\beta}\,\vec{y_3}}$$

$$\boxed{P_{lame \to 3} = F\,b\,\dot{\beta}\cos\beta.}$$