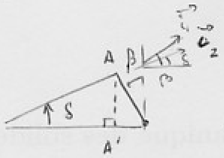
Examen de l'année 09-10

Examen éolienne

12h05 1)



$$\left\{\begin{array}{l} \sin\delta = \dfrac{AA'}{e} \\ \cos\beta = \dfrac{AA'}{a} \end{array}\right. \qquad AA' = a\cos\beta$$

$$\boxed{\sin\delta = \frac{a}{e}\cos\beta}$$

2) le problème est plan, dans le plan $(B, \vec{i}, \vec{j})$

12h08 3) On isole la poutre ABC

Bilan des actions

Force en A $\quad \left\{\mathcal{T}_1\right\} = \left\{\begin{array}{c} -F_1\vec{i_2} \\ \vec{0k} \end{array}\right\}_A$

Liaison appui simple $\quad \left\{\mathcal{T}_2\right\} = \left\{\begin{array}{c} R_{2i}\vec{i} + R_{2j}\vec{j} \\ \vec{0k} \end{array}\right\}_B$

Nacelle $\quad \left\{\mathcal{T}_3\right\} = \left\{\begin{array}{c} -m_1 g\vec{j} \\ \vec{0k} \end{array}\right\}_C$

mât $\quad \left\{d\mathcal{T}_4\right\} = \left\{\begin{array}{c} -\rho_\ell\, ds_2\, g\vec{j} \\ \vec{0k} \end{array}\right\}_P \quad$ avec $\vec{BP} = s_2\vec{i_1}$

avec ρ_ℓ la masse linéique en kg m^{-1}

$$\rho_\ell = \frac{m_2}{2b}$$

L'équilibre donne $\quad \sum\left\{\mathcal{T}\right\} = \left\{0\right\}$

$$\left\{\begin{array}{l} -F_1\vec{i_2} \\ -F_1\vec{i_2}\wedge -a\vec{j_1} \end{array}\right\}_B + \left\{\begin{array}{c} R_{2i}\vec{i} + R_{2j}\vec{j} \\ \vec{0k} \end{array}\right\}_B + \left\{\begin{array}{l} -m_1 g\vec{j} \\ -m_1 g\vec{j}\wedge -2b\vec{i_1} \end{array}\right\}_B$$

$$+ \int_0^{2b} \left\{\begin{array}{l} -\rho_\ell\, ds_2\, g\vec{j} \\ -\rho_\ell\, ds_2\, g\vec{j}\wedge -s_2\vec{i_1} \end{array}\right\}_B = \left\{0\right\}$$

l'équation de moment autour de $B\vec{k}$ donne

$$F_1 a \sin\left(-\delta + \frac{\pi}{2} + \beta\right) + 2b\, m_1 g \sin\left(-\frac{\pi}{2} + \beta\right)$$

$$+ \int_0^{2b} \rho_\ell\, g \sin\left(-\frac{\pi}{2} + \beta\right) s_2\, ds_2 = 0$$

$$F_1 = \frac{2b\, m_1 g \sin\left(\beta - \frac{\pi}{2}\right) + \rho_\ell\, g\, 2b^2 \sin\left(\beta - \frac{\pi}{2}\right)}{a \sin\left(\beta + \frac{\pi}{2} - \delta\right)}$$

(2)

$$\boxed{F_1 = 2bg\,(-\cos\beta)\,\frac{m_1 + \frac{m_2}{2}}{a\,\cancel{\sin}\cos(\beta-\delta)}}$$

12h22 4) les équations de résultante donnent

$$\begin{cases} -F_1\cos\delta + R_{2i} = 0 \\ -F_1\sin\delta + R_{2j} - m_1 g - \rho_\ell\, g\, 2b = 0 \end{cases}$$

$$\begin{cases} R_{2i} = F_1\cos\delta \\ R_{2j} = F_1\sin\delta + g\,(m_1 + m_2) \end{cases}$$

F_1 connu par la question 3

le système est isostatique

5) On oriente la poutre de A vers B

On note $\overrightarrow{AH_1} = s_1\,(-\vec{j}_1)$

$$\left\{\mathcal{T}_{eff\,int\,H_1}\right\} = -\left\{\mathcal{T}_{seg-}\right\}$$

$$= -\left\{\mathcal{T}_1\right\}$$

$$= \begin{Bmatrix} F_1\,\vec{i}_2 \\ 0\,\vec{k} \end{Bmatrix}_A$$

$$= \begin{Bmatrix} F_1\,\vec{i}_2 \\ F_1\vec{i}_2 \wedge -s_1\vec{j}_1 \end{Bmatrix}_{H_1}$$

$$= \begin{Bmatrix} F_1\,\vec{i}_2 \\ -F_1 s_1\,\sin\left(-\delta+\frac{\pi}{2}+\beta\right)\vec{k} \end{Bmatrix}_{H_1}$$

$$= \begin{Bmatrix} F_1\,\vec{i}_2 \\ -F_1 s_1\cos(\beta-\delta)\,\vec{k} \end{Bmatrix}_{H_1}$$

12h30 6) le repère local en H_1 est tel que

$$\begin{cases} \vec{x} = -\vec{j}_1 \\ \vec{y} = \vec{i}_1 \\ \vec{z} = \vec{k} \end{cases}$$

$$\left\{\mathcal{T}_{eff\,int\,H_1}\right\} = \begin{Bmatrix} F_1\cos(\beta-\delta)\,\vec{i}_1 + F_1\sin(\beta-\delta)\,\vec{j}_1 \\ -F_1 s_1\cos(\beta-\delta)\,\vec{z} \end{Bmatrix}_{H_1}$$

③

$$\left\{\mathcal{T}_{eff\,int_{H_1}}\right\} = \left\{\begin{array}{l} -F_1 \sin(\beta-\delta)\ \vec{x} + F_1 \cos(\beta-\delta)\ \vec{y} \\ -F_1\ s_1 \cos(\beta-\delta)\ \vec{z} \end{array}\right\}_{H_1}$$

$$\left\{\begin{array}{lll} \text{effort normal} & N_1 = & -F_1 \sin(\beta-\delta) \\ \text{effort tranchant} & T_1 = & F_1 \cos(\beta-\delta) \\ \text{moment fléchissant} & M_{f_{3_1}} = & -F_1\, s_1 \cos(\beta-\delta) \end{array}\right.$$

30'

12h35

13h20

7)
$$\left\{\mathcal{T}_{eff\,int_{H_2}}\right\} = \Sigma \left\{\mathcal{T}_{seg+}\right\}$$

$$= \left\{\mathcal{T}_3\right\} + \int_{s_2}^{2b} \left\{d\mathcal{T}_4\right\}$$

$$= \left\{\begin{array}{l} -m_1 g\ \vec{J} \\ 0\ \vec{k} \end{array}\right\}_C + \int_{s_2}^{2b} \left\{\begin{array}{l} -\rho_\ell\, g\, ds_P\ \vec{J} \\ 0\ \vec{k} \end{array}\right\}_P$$

$$= \left\{\begin{array}{l} -m_1 g\ \vec{J} \\ -m_1 g \vec{J} \wedge (2b-s_2)(-\vec{i_1}) \end{array}\right\}_{H_2} + \int_{s_2}^{2b} \left\{\begin{array}{l} -\rho_\ell\, g\, ds_P\ \vec{J} \\ -\rho_\ell\, g\, ds_P \vec{J} \wedge \cancel{2b}\,(s_P-s_2)(-\vec{i_1}) \end{array}\right\}_{H_2}$$

$$= \left\{\begin{array}{l} \left[-m_1 g - \rho_\ell\, g\,(2b-s_2)\right] \vec{J} \\ \left[+m_1 g\,(2b-s_2)\ \sin\left(-\frac{\pi}{2}+\beta\right) \right. \\ \left. \quad + \rho_\ell\, g\ \sin\left(-\frac{\pi}{2}+\beta\right) \int_{s_2}^{2b} (s_P-s_2)\, ds_P \right] \vec{k} \end{array}\right\}_{H_2}$$

$$= \left\{\begin{array}{l} -g\left[m_1 + m_2\left(1-\frac{s_2}{2b}\right)\right] \vec{J} \\ -g \cos\beta \left[m_1 (2b-s_2) + \frac{m_2}{2b} \int_0^{2b-s_2} u\, du\right] \vec{k} \end{array}\right\}_{H_2}$$

$$= \left\{\begin{array}{l} -g\left[m_1 + m_2\left(1-\frac{s_2}{2b}\right)\right] \vec{J} \\ -g \cos\beta \left[m_1 (2b-s_2) + \frac{m_2}{4b} (2b-s_2)^2\right] \vec{k} \end{array}\right\}_{H_2}$$

13h29

8) le repère local est tel que

$$\left\{\begin{array}{l} \vec{x} = \vec{i_1} \\ \vec{y} = \vec{J_1} \\ \vec{z} = \vec{k} \end{array}\right.$$

$$\left\{\mathcal{T}_{eff\,int}\right\} = \left\{\begin{array}{l} -g\left[m_1 + m_2\left(1-\frac{s_2}{2b}\right)\right] \left(\sin\beta\, \vec{x} + \cos\beta\, \vec{y}\right) \\ -g\cos\beta \left[m_1(2b-s_2) + \frac{m_2}{4b}(2b-s_2)^2\right] \vec{z} \end{array}\right\}_{H_2}$$

13h32 8) effort normal $N_2 = -g(m_1 + m_2(1 - \frac{s_2}{2b}))\sin\beta$

effort tranchant $T_2 = -g(m_1 + m_2(1 - \frac{s_2}{2b}))\cos\beta$

moment fléchissant $M_{f_{32}} = -g\cos\beta\left[m_1(2b - s_2) + \frac{m_2}{4b}(2b - s_2)^2\right]$

13h33 9) Pour une poutre en flexion - traction

$$|\sigma_M(\xi)| = \left|\frac{N}{S}\right| + \left|\frac{M_{f_3}\tilde{y}}{I_{H_3}}\right|$$

avec $S = \frac{\pi}{4}\left(d^2 - (d - 2e')^2\right)$

$\tilde{y} = \frac{d}{2}$

$I_{H_3} = \frac{\pi}{32}\left(d^4 - (d - 2e')^4\right)$

l'effort normal et le moment fléchissant sont maximaux pour $s_2 = 0$ donc a proximité de B

$$\sigma_M = \frac{g(m_1 + m_2)\sin\beta}{\frac{\pi}{4}\left(d^2 - (d - 2e')^2\right)} + \frac{g\cos\beta\left[m_1 2b + m_2 b\right] d\, 16}{\pi\left(d^4 - (d - 2e')^4\right)}$$

13h39 10) Dans le repère $(B, \vec{\imath}_1, \vec{\jmath}_1)$ le point B ne tourne pas

$\omega_B = 0$ et est fixe $v_B = 0$.

On utilise la formule de Bresse

$$\begin{cases} \vec{v}_B = \vec{v}_A + \vec{\omega}_A \wedge \vec{AB} + \int_0^a \frac{N_1}{ES}\vec{x}\, ds_1 + \int_0^a \frac{M_{f_{31}}}{EI_{H_{3}}}\vec{3} \wedge \vec{PB}\, ds_p \\ \vec{\omega}_B = \vec{\omega}_A + \int_0^a \frac{M_{f_{31}}}{EI_{H_3}}\vec{3}\, ds_p \end{cases}$$

$$\vec{\omega}_A = -\int_0^a \frac{M_{f_{31}}}{EI_{H_3}}\vec{3}\, ds_p$$

$$\vec{v}_A = \int_0^a \frac{M_{f_{31}}}{EI_{H_3}}\vec{3}\, ds_p \wedge (-a)\vec{\jmath}_1$$

$$- \int_0^a \frac{N_1}{ES}\vec{x}\, ds_1$$

$$- \int_0^a \frac{M_{f_{31}}}{EI_{H_3}}\vec{3} \wedge \vec{PB}\, ds_p$$

$$\vec{v_A} = + \frac{a F_1 \cos(\beta-\delta)}{EI_{H_3}} \frac{a^2}{2} (-\vec{i_1})$$

$$+ \frac{1}{ES} F \sin(\beta-\delta) a (-\vec{j_1})$$

$$\frac{F_1 \cos(\beta-\delta)}{EI_{H_3}} \int_0^a s_1 (a - s_1) \vec{i_1} \, ds_1$$

$$= \vec{i_1} \left[\frac{F_1 \cos(\beta-\delta)}{EI_{H_3}} \left[-\frac{a^3}{2} + \frac{a^3}{2} - \frac{a^3}{3} \right] \right]$$

30' + 30'

13h50) $$+ \vec{j_1} \left[-\frac{F \sin(\beta-\delta) a}{ES} \right]$$

13h53) 11) $$d_0 = \left[a^2 + b^2 \right]^{1/2}$$

12) $\cancel{d_1 = \ldots}$

$$\vec{A'D} = \vec{A'A} + \vec{AD}$$

$$= -\vec{v_A} + -a\vec{j_1} + b\vec{i_1}$$

$$= \left[b + \frac{a^3}{3} \frac{F_1 \cos(\beta-\delta)}{EI_{H_3}} \right] \vec{i_1}$$

$$+ \left[-a + a \frac{F \sin(\beta-\delta)}{ES} \right] \vec{j_1}$$

30' + 30' + 5'

13h58) $$d_1 = \left[\left[a \left(1 - \frac{F \sin(\beta-\delta)}{ES} \right) \right]^2 + \left[b + \frac{a^3}{3} \frac{F_1 \cos(\beta-\delta)}{EI_{H_3}} \right]^2 \right]^{1/2}$$

<u>haubanage.</u>

10h37 1) D —— H_4 —— A —→ F_c → $\vec{i_3}$

$$\left\{ \mathcal{T}_{\text{eff int}} \right\} = \left\{ \begin{matrix} F_c \vec{i_2} \\ 0 \vec{k} \end{matrix} \right\}_A = \left\{ \begin{matrix} F_c \vec{i_2} \\ 0 \vec{k} \end{matrix} \right\}_{H_4}.$$

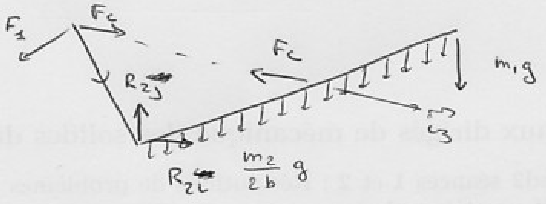
effort normal $N = F_c$

$$\varepsilon_x = \frac{N}{E_c S_c} = \frac{F_c}{E_c S_c}$$

$$a_2 = \int_0^{\sqrt{a^2+b^2}} \varepsilon_x \, ds = \frac{F_c \sqrt{a^2+b^2}}{E_c S_c}$$

10h40)

10h48) 2)



10h50) 3) en H_1 : oui car $\left\{ \mathcal{T}_{eff\,int\,H_1} \right\} = -\left\{ \mathcal{T}_1 \right\} - \left\{ \begin{matrix} F_c \vec{y_3} \\ 0 \vec{k} \end{matrix} \right\}_A$

en H_2 : oui car $\left\{ \mathcal{T}_{eff\,int\,H_2} \right\} = \left\{ \mathcal{T}_3 \right\} + \int_{H_2}^{C} \left\{ d\mathcal{T}_4 \right\} + \left\{ \begin{matrix} -F_c \vec{y_3} \\ 0 \vec{k} \end{matrix} \right\}_D$

en H_3 : non car $\left\{ \mathcal{T}_{eff\,int\,H_3} \right\} = \left\{ \mathcal{T}_3 \right\} + \int_{H_3}^{C} \left\{ d\mathcal{T}_4 \right\}$

10h52) Il faut que l'allongement du cable soit égal à la variation de distances entre A et D.

7'

$$a_2(F_c) = d_1(F_c) - \sqrt{a^2+b^2}$$

10h53)

Examen de 67' pour rédiger la correction.

note maximale $= \frac{67}{40} \times 20 = 33,5$

notation sur 33,5 points